MA126696

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REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. AD. A126 676	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
SOME COMPLEXITY RESULTS ABOUT PACKET RADIO NETWORKS	Technical
	6. PERFORMING ORG. REPORT NUMBER LIDS-P-1284

7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
Erdal Arikan	ARPA Order No. 3045/5-7-57 ONR/N00014-75-C-1183

9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
M.I.T. Laboratory for Information and Decision Systems Cambridge, MA 02139	Program Code No. 5T10 ONR Identifying No. 049-383

11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209	March, 1983
	13. NUMBER OF PAGES 26

14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)	15. SECURITY CLASS. (of this report)
Office of Naval Research Information Systems Program Code 437 Arlington, Virginia 22217	Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release: distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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The analysis includes a linear programming formulation of TDMA (time-division-multi-access) schemes for PRNs and NP-completeness results for some

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S/N 0102-LF-014-6601

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SOME COMPLEXITY RESULTS ABOUT PACKET RADIO NETWORKS*

by

Erdal Arikan**

ABSTRACT

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given point in the capacity region of a packet radio network (PRN) is NP-
complete. The capacity region is the set of all feasible sets of average
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moving the data through the network so that the desired rates are satisfied.

The analysis includes a linear programming formulation of TDMA (time-
division-multi-access) schemes for PRNs and NP-completeness results for some
other related problems. Implicit in the analysis is the optimality of
TDMA schemes in terms of achieving a given set of link traffic rates.

I. INTRODUCTION

A packet radio network (PRN) is a collection of geographically distributed and possibly mobile users which share a common radio channel for transmitting and receiving messages to and from each other. Among the distinctive features of a PRN are the burstiness of message traffic and the fact that not all users are necessarily within the line of sight of one another. PRNs present problems which are different in nature and more difficult than those encountered in wire or satellite networks, [1]-[4]. For a complete survey of the issues concerning PRNs, we refer to Kahn et al., [1].

We are interested here mainly in the capacity region of a PRN. We seek an algorithmic solution to the problem of determining whether a given point belongs to the capacity region of a PRN and in fact show that this decision problem is NP-complete. This result is obtained through a formulation which allows one to identify the capacity region of a PRN with the set of achievable origin-to-destination (o-d) traffic rates under perfect scheduling with complete information about network topology and desired rates.

II. A PRN MODEL

As the first step toward the formulation of the problem we develop a simple PRN model. A PRN is represented by a directed graph $G = (N,A)$ such that each user of the network is associated with a node in N and there exists a link (a,b) in A iff node b is within the transmission range of node a. G is a simple graph, i.e. G contains no self loops or multiple links from one user to another.

When a node transmits a packet, it uses the entire bandwidth that is available. In terms of the graph representation, this is to say that at any time a node can send at most one packet over any of its outgoing links. In fact, because of the broadcast nature of the network, when a node sends a packet, this packet gets sent over all of its outgoing links at the same time. As a result, a transmitted packet reaches all neighbors of the transmitting node whether or not each neighbor is an intended receiver for that packet. To make this distinction clear, we will say that a packet is transmitted over link (a,b) only if node b is an intended receiver for this packet transmitted by node a.

A transmitted packet may have several intended receivers if the network protocols so desire. However, we shall restrict the data traffic with respect to the number of (final) destinations by assuming that each packet in the network has only one destination.

An (a,b)-transmission is said to be successful if the packet transmitted over link (a,b) is received correctly by node b. In this model the only cause of unsuccessful packet transmissions is interference between different transmissions. Interference

can be described as a binary relation on the set of links: $(c,d) \epsilon A$ interferes
with $(a,b) \epsilon A$ iff $a \neq c$ and either $c = b$ or else $(c,b) \epsilon A$. (Figure 1 a,b,c.)

We say that $(c,d) \epsilon A$ conflicts with $(a,b) \epsilon A$ iff either (c,d) interferes
with (a,b) or $a = c$ and $b \neq d$. (Figure 1.d.) The latter case corresponds
to the exclusion of simultaneous transmission of two or more different
packets by the same node. We define C_{ab} to be the set of all links that
conflict with link (a,b); thus, $C_{ab} = \{(c,d) \epsilon A: c \neq a, (c,b) \epsilon A\} \cup \{(a,d) \epsilon A:$
$d \neq b\} \cup \{(b,d) \epsilon A\}$. The significance of C_{ab} is that a transmission over
(a,b) is successful iff no link in C_{ab} attempts to transmit another packet
simultaneously.

We assume that the average desired traffic rates are fixed for each
o-d pair and denote the collection of all o-d rates by a column vector $\vec{r}$
whose $(x,y)^{th}$ row, r_{xy}, is the desired rate for o-d pair $(x,y) \epsilon$ NxN, $x \neq y$.

If there exist network protocols which satisfy the desired rates for
each o-d pair, then these o-d rates are called feasible. The capacity region
$C(G)$ of a PRN G is the set of all feasible o-d rate vectors.

The problem of major interest here is to determine whether a given
o-d rate vector $\vec{r}$ belongs to the capacity region $C(G)$ of a given PRN G.
The $\vec{r}$-feasibility problem, as this problem is called, is difficult to
formulate in a general setting due to the vague notion of existence of
certain network protocols. For this reason, we initially consider a re-
stricted problem by assuming that 1) $r_{xy} = 0$ unless $(x,y) \epsilon A$ and 2) each
(x,y)-packet (i.e. a packet with origin x and destination y) is sent
directly over link (x,y) whenever $(x,y) \epsilon$ A.

Under restrictions 1 and 2, the desired flow rate f_{xy} across link

(x,y) equals r_{xy} for each $(x,y) \epsilon A$. We represent link flow rates by a column $|A|$-vector $\vec{f}$, and use the term $\vec{f}$-feasibility problem for the restricted version of the $\vec{r}$-feasibility problem.

The feasibility of $\vec{f}$ depends only on the timing and duration of individual transmissions; i.e. it involves no routing decisions. This is a scheduling problem where the set of rules determining the schedule is usually called a multi-access scheme. In the next section, we shall analyze the $\vec{f}$-feasibility problem under TDMA schemes which have the desirable property that if a given $\vec{f}$ is not feasible under TDMA then it is not feasible under any other multi-access scheme.

As a final simplifying assumption, each link in the network will have the same capacity. Link traffic rates will be normalized with respect to this capacity so that $\vec{0} \leq \vec{f} \leq \vec{1}$ for any feasible $\vec{f}$.

III. TDMA SCHEMES

Consider a variable slot length TDMA scheme where all nodes are synchronized so that each transmission slot as perceived by different users starts and ends simultaneously. With each slot associate a column L-vector $\vec{t}$ where L is the number of links in the network and $t_i = 1$ if link i transmits in that slot; $t_i = 0$ otherwise, for $i = 1, \ldots, L$.

The transmission vectors that we consider here are conflict-free in the sense that whenever $t_i = 1$ for some link i, $t_j = 0$ for all $j \in C_i$.

The set of all links used by a transmission vector $\vec{t}$, i.e. the set $\{i \in A : t_i = 1\}$, is called the transmission set of $\vec{t}$. A transmission set is called maximal if it is not contained in any other transmission set. A maximal transmission vector is one whose transmission set is maximal. As the PRN in Figure 2 shows, the number of maximal transmission vectors need not be polynomially bounded in the number of links in the network. This fact has important consequences in terms of the complexity of the feasibility problems as will be explored later.

Let $\vec{t}_1, \vec{t}_2, \ldots, \vec{t}_K$ be an ordering of all transmission vectors and let T be the LxK matrix whose i^{th} column is $\vec{t}_i$ $(i = 1, \ldots, K)$. The particular ordering of the transmission vectors as columns of T is not important in this formulation, so T is treated as if it is unique and called the transmission matrix.

With no loss of generality, we can consider variable slot length TDMA schemes in which each transmission vector is used only once in a frame. We let $x_i \geq 0$ be the slot length of time in a frame for which transmission vector $\vec{t}_i$ is used. Also with no loss of generality, we let

the frame length to be 1. Clearly, $\vec{f}$ is feasible under TDMA iff $1 \geq \min$ $\vec{1} \cdot \vec{x}$ subject to $T\vec{x} = \vec{f}$ and $\vec{x} \geq \vec{0}$.

The fact that the TDMA-$\vec{f}$-feasibility problem can be formulated as a linear program does not directly guarantee its solution in time polynomially bounded in L because, as we have shown, the number of columns of T need not be bounded polynomially in L even when only maximal transmission vectors are counted.

Before giving the complexity results about the feasibility problems, we need to define them in a more precise way.

A TDMA scheme is a three-tuple $\langle G,T,\vec{x}\rangle$ where G is a PRN, T is the transmission matrix of G and $\vec{x}$ is a column K-vector such that $\vec{1} \cdot \vec{x} \leq 1$ and $\vec{x} \geq \vec{0}$.

FF (TDMA-$\vec{f}$-feasibility problem)

Instance: $\langle G,\vec{f}\rangle$ where G is a PRN and $\vec{f}$ is a column L-vector (one element for each link) with $\vec{0} \leq \vec{f} \leq \vec{1}$.

Question: Does there exist a TDMA scheme $\langle G,T,x\rangle$ such that $Tx = \vec{f}$?

RF ($\vec{r}$-feasibility problem)

Instance: $\langle G,\vec{r}\rangle$ where G is a PRN and $\vec{r}$ is a non-negative column vector with one element for each o-d pair in G.

Question: Is it true that $\vec{r}\in C(G)$ where C(G) is the capacity region of PRN G?

The main result is the following:

Theorem. FF and RF are NP-complete.

The proof is given in the appendix. The NP-completeness of FF is a consequence of the fact that T may have exponentially many columns in L.

IV. DISCUSSION OF RESULTS

The NP-completeness of FF and RF is a discouraging result because PRNs need fast and reliable algorithms that can adapt to changes in data traffic and network topology for their proper and efficient operation.

Further results on the complexity of the feasibility problems are given in [5] where it is shown that even approximate solutions to FF and RF cannot be obtained in polynomial time unless similar approximation algorithms exist for the CLIQUE problem [6], which is an extensively studied problem for which no polynomial time approximation algorithm is known.

The reduction proofs in the appendix make it clear that NP-complete prevails also in cases where slots are constrained to be fixed in size (i.e. $x_i \in \{0,c\}$ where c is constant) or larger than a fixed length in duration (i.e. $x_i = 0$ or $x_i \geq c$ for some constant c). The feasibility problems remain NP-complete for the subclass of PRNs $G(N,A)$ which satisfy $(a,b) \in A \Longrightarrow (b,a) \in A$ for all $(a,b) \in A$.

Finally let us emphasize that the NP-completeness of RF is a general result where the routing and multi-access schemes are not restricted in any way. As can be seen in the appendix, the proof of this result is made possible by the fact that if a set of link flow rates $\vec{f}$ is feasible under any multi-access scheme, than it is feasible under TDMA. To see this, suppose $\vec{f}$ is feasible under some multi-access scheme, i.e. there exists a finite time instant α such that the desired traffic rates are satisfied during the time interval $[0,\alpha]$. Let $s(t)$ be the set of stations in transmission at time $t \in [0,\alpha]$. For fixed t, if $s(t)$ is a (conflict-free) transmission set, then leave $s(t)$ as it is; otherwise find a link i whose transmission is destroyed by interference and delete it from $s(t)$, i.e. let $s(t) \leftarrow s(t) \backslash \{i\}$.

Do this until $s(t)$ contains no conflicting links for all $t \varepsilon [0,\alpha]$. Any transmission that was successful remains successful at the end of this process; therefore, the desired rates are still satisfied under the new scheme which allows only non-conflicting sets of links to transmit at any instant of time in the interval $[0,\alpha]$. We now let x_i be the average amount of time transmission vector $\vec{t}_i$ is used in the interval $[0,\alpha]$ to obtain a TDMA scheme under which $\vec{f}$ is feasible.

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APPENDIX

Notation.

We use a shorthand notation for the cartesian product of an arbitrary set V and an index set I which requires some explanation. For $x \epsilon V$, $i \epsilon I$, we denote $(x,i) \epsilon$ VxI by x_i and denote $\{(x,i) : x \epsilon V\}$ by $V_i = \{x_i : x \epsilon V\}$. We refer to V_i as the ith copy of V.

1. NP-Completeness of FF

To prove that FF is NP-complete we first show that FF ∈ NP and then give a polynomial reduction from a related NP-complete problem.

Lemma 1. FF ∈ NP

Proof. If $\vec{f}$ is feasible, then the linear program of section III has a basic feasible solution, i.e., there exists L linearly independent transmission vectors $\vec{t}_{i_1}, \vec{t}_{i_2},\ldots,\vec{t}_{i_L}$ such that $B\,\vec{x}_B = \vec{f}$ and $\vec{1}\cdot\vec{x}_B \leq 1$, where B is the LxL basis matrix whose j^{th} column is $\vec{t}_{i_j}$. Therefore, a concise certificate for a YES instance of FF is the basis matrix B. Note that B^{-1} can be computed in time polynomially bounded in L and $\vec{x}_B$ can be computed in time polynomially bounded in L and the size of the input $<G,\vec{f}>$. ||

Consider now the following problem which will be shown to be NP-complete and polynomially transformable to FF, thereby completing the proof that FF is NP-complete.

Definition. The Transmission Set Cardinality Problem (TSC)

Instance: $<G,D,k>$ where G = (N,A) is a PRN, D ⊂ A is a set of links, and k is a positive integer not greater than |D|.

Question: Does there exist a transmission set S of G such that $|S\cap D| = k$?

Lemma 2. TSC is NP-complete.

Proof. The reduction is from the CLIQUE problem: Given $<H,k>$ where H = (V,E) is an undirected graph and k is integer with $0 < k \leq |V|$, does H contain a clique of size k, i.e. a complete subgraph with k nodes? CLIQUE is NP-complete [6] and the following algorithm transforms CLIQUE to FF.

Algorithm Al.

Input: $<H,k>$ (an instance of CLIQUE),

Output: $<G,D,k>$ (an instance of FF).

G = (N,A) is related to H = (V,E) as follows.

$N = V_1 \cup V_2$, $A = \{(a_1,b_2),(b_2,a_1) : a \varepsilon V, b \varepsilon V, (a,b) \notin E\}$, $D = \{(a_1,a_2) : a \varepsilon V\}$.

The algorithm is illustrated in Figure Al by an example.

If Q is a clique in H with $|Q| = k$, then $S = \{(a_1,a_2) : a \varepsilon Q\}$ is a transmission set of G and $|S \cap D| = k$. Conversely, if S is a transmission set of G with $|S \cap D| = k$, then $Q = \{a \varepsilon V : (a_1,a_2) \varepsilon \ S \cap D\}$ is a clique of size k in H. Therefore, $<H,k>$ is a YES instance of CLIQUE iff $<G,D,k>$ is a YES instance of TSC. To complete the proof that TSC is NP-complete we note that a concise certificate for a YES instance of TSC is the set S. //

To show that TSC is reducible to FF we need a special construction which is introduced next.

Definition. The mth power of G = (N,A) is $G^m = (N^m,A^m)$ where $N^m = \bigcup_{i=1}^{m} N_i$, $A^m = \bigcup_{i=1}^{m} \bigcup_{j=1}^{m} A_{ij}$,

$$A_{ii} = \{(a_i,b_i) : a_i \varepsilon N_i, b_i \varepsilon N_i, a_i \neq b_i\} , \quad i = 1,\ldots,m,$$

$$A_{ij} = \{(a_i,b_j) : a_i \varepsilon N_i, b_j \varepsilon N_j, a = b \text{ or } (a,b) \varepsilon A\} , \quad i \neq j,$$
$i = 1.,\ldots,m, j = 1,\ldots,m.$

In Figure A2, G^2 is shown as an example for a small graph G. Note that (N_1,A_{11}) and (N_2,A_{22}) are complete directed graphs. If we regard G and G^2 as PRNs, then it can be seen that whenever (x,y) and (u,v) are non-conflicting links in G, (x_1,y_1) and (u_2,v_2) are non-conlficting in G^2.

Lemma 3. Let $G = (N,A)$ be a PRN and $G^m = (N^m, A^m)$ be the mth power of G. For any two links $(x,y) \varepsilon A$ and $(u,v) \varepsilon A$, (x,y) does not conflict with (u,v) in G iff (x_i, y_i) does not conflict with (u_j, v_j) in G^m for all i and j such that $i \neq j$ and $1 \leq i, j \leq m$.

Proof. Suppose (x,y) does not conflict with (u,v). Let i and j be such that $i \neq j$ and $1 \leq i, j \leq m$. By the hypothesis, $x \neq v$ and $(x,v) \notin A$. Hence $(x_i, v_j) \notin A^m$ and it follows that (x_i, y_i) does not conflict with (u_j, v_j).

Suppose (x,y) conflicts with (u,v). Then either $x = v$ or $(x,v) \varepsilon A$. In each case $(x_i, v_j) \varepsilon A_{ij}$ for all i,j such that $i \neq j$ and $1 \leq i, j \leq m$; hence (x_i, y_i) conflicts with (u_j, v_j).//

Lemma 4. Let $<G,D,k>$ be an instance of TSC and $D^k = \bigcup\limits_{i=1}^{k} D_i$. The following statements are equivalent:

(i)   There exists a transmission set S of G such that $|S \cap D| = k$.

(ii) There exists a transmission set S^k of G^k such that $|S^k \cap D^k| = k$.

Proof. (i) $\implies$ (ii): Let $S \cap D = \{\ell_1, \ell_2, \ldots, \ell_k\}$ and let ℓ_{ij} be the copy of ℓ_i in A_{jj}, i.e., if $\ell_i = (a,b)$, then $\ell_{ij} = (a_j, b_j)$ for $1 \leq i \leq k$ and $1 \leq j \leq k$. By Lemma 3, $S^k = \{\ell_{ii} : i = 1, \ldots, k\}$ is a transmission set of G^k and $|S^k \cap D^k| = k$.

(ii) $\implies$ (i): S^k must contain exactly k links with one link from each D_i $(i = 1, \ldots, k)$ because any two distinct links in D_i conflict with each other. Therefore $S = \{(a,b): (a_i, b_i) \varepsilon S^k$ for some $i \varepsilon \{1, \ldots, k\}\}$ has k elements and is a transmission set of G. Since $S \subset D$, $S \cap D$ also has k elements.//

Finally, we consider the algorithm that transforms TSC to FF.

Algorithm A2.

Input: $\langle G, D, k \rangle$ (an instance of TSC)

Output: $\langle G', f' \rangle$ (an instance of FF) where $G' = (N', A')$ and $G = (N, A)$

are related as follows:

$$N' = N^k \cup P \cup Q$$

$$N^k = \bigcup_{i=1}^{k} N_i$$

$$P = \{p_1, p_2, \ldots, p_k\}$$

$$Q = \{q_1, q_2, \ldots, q_k\}$$

$$A' = A^k \cup Y \cup Z$$

$$A^k = \bigcup_{i=1}^{k} \bigcup_{j=1}^{k} A_{ij}$$

$$A_{ii} = \{(a_i, b_i) \in N_i \times N_i : a \neq b\} \quad , \quad i = 1, \ldots, k,$$

$$A_{ij} = \{(a_i, b_j) \in N_i \times N_j : a = b \text{ or } (a,b) \in A\} \ , \quad i \neq j,$$

$$i = 1, \ldots, k, \ j = 1, \ldots, k,$$

$$Y = (P \times Q) \cup (Q \times P)$$

$$Z = \bigcup_{\substack{i=1 \\ j \neq i}}^{k} \bigcup_{j=1}^{k} Z_{ij}$$

$$Z_{ij} = \{p_i\} \times N_j \cup N_j \times \{p_i\} \quad , \ j \neq i, \ j = 1, \ldots, k, \ i = 1, \ldots, k$$

$$f'_{xy} = \begin{cases} \delta & ; \ (x,y) \in D^k \\ (|D|-1) \, \delta & ; \ (x,y) \in C \\ 0 & ; \ \text{otherwise,} \end{cases}$$

where $\delta = \dfrac{1}{1 + k(|D|-1)}$,

$$D^k = \bigcup_{i=1}^{k} D_i, \quad C = \{(p_i, q_i) \in P \times Q : i = 1, \ldots, k\}.$$

Note that the kth power of G, (N^k, A^k), is part of G'. The complete structure of G' is illustrated in Figure A3 for the case of $|D| = 4$ and $k = 3$. The part of G' enclosed in the rectangle is G^3. As seen in the figure, (P,Q,Y) is a complete bipartite graph, i.e. $(x,y) \epsilon Y$ iff ($x \epsilon P$ and $y \epsilon Q$) or ($x \epsilon Q$ and $y \epsilon P$). So, transmission sets of G' can have at most one link in common with Y. Likewise, $(\{p_i\}, N_j, Z_{ij})$ is a complete bipartite graph for $i \neq j$ and if a transmission set S' contains a link of the form (p_i, x) for some $x \epsilon N'$, then $S' \cap A_{ij} = \emptyset$. In fact, if $(p_i, x) \epsilon S'$, then S' can have at most two elements, the other one being any link in A_{ii}. The reader should verify the above statements for himself before proceeding with the following lemma.

Lemma 5. Let $<G,D,k>$ be an input of algorithm A2 and $<G',f'>$ be the corresponding output. Then, $<G,D,k>$ is a YES instance of TSC iff $<G',f'>$ is a YES instance of FF.

Proof. Suppose $<G,D,k>$ is a YES instance of FF. By Lemma 4, there exists a transmission set S' of G' such that $S' \subset D^k$ and $|S'| = k$. Thus, if we use S' for δ time units, then we can satisfy the desired flow rates of k links in G, with one link from each set D_i ($i = 1,\ldots,k$). There remains k-1 links in each D_i ($i = 1,\ldots,k$) which are not included in S', so let $D_i \setminus S' = \{(a_{ij}, b_{ij}) : j = 1,\ldots k-1\}$ be a labelling of such links. Define transmission sets $S_{ij} = \{(a_{ij}, b_{ij}), (p_i, q_i)\}$ for $i = 1,\ldots,k$, $j = 1,\ldots,k-1$. Now, it can be verified easily that $\vec{f'}$ is feasible under a TDMA scheme which uses the transmission sets S' and S_{ij} ($i = 1,\ldots,k$, $j = 1,\ldots,k-1$) each for δ fraction of the time.

Suppose that $\vec{f'}$ is feasible under TDMA. Observe that any TDMA scheme

under which $\vec{f}'$ is feasible must spend at least $1-\delta$ fraction of the time in satisfying the traffic assignments on links in C. In the remaining δ fraction of the time at least k links in D^k must be allowed to transmit simultaneously. This implies that there exists a transmission set S' of G' with $|S' \cap D^k| = k$. By Lemma 4, there exists a transmission set S of G with $|S \cap D| = k$. This completes the proof of the lemma and the NP-completeness of FF.

2. NP-Completeness of RF

Define the total traffic rate associated with node x of a PRN $G = (N,A)$ as $R_x = \sum_{y \in N} r_{xy} + \sum_{y \in N} r_{yx}$. R_x is the minimum fraction of time node x is busy with receiving or transmitting messages which either originate at x or terminate at x. Clearly, if $<G,\vec{r}>$ is a YES instance of RF, then we must have $R_x \leq 1$ for all $x \in N$. The following algorithm makes use of this fact in reducing FF to RF.

Algorithm A3.

Input: $<G,\vec{f}>$ (an instance of FF)

Output: $<G',\vec{r}'>$ (an instance of RF) where $G' = (N',A')$ and
$G = (N,A)$ are related as follows:

$$N' = N \cup N_1 \cup N_2$$

$$A' = A \cup B \cup C , \quad B = \{(a_1,a_2) : a \in N\}$$

$$C = \{(a,a_2), (a_2,a), (a,a_1), (a_1,a), (a_2,a_1) : a \in N\}$$

$$r'_{ab} = f_{ab} \qquad\qquad ; \quad (a,b) \in A$$

$$r'_{a_1 a_2} = 1 - \sum_b (f_{ab} + f_{ba}) \qquad ; \quad (a_1,a_2) \in B$$

$$r'_{xy} = 0 \qquad\qquad ; \quad (x,y) \in C \text{ or } (x,y) \notin A'$$

The algorithm is illustrated in Figure A4.

Lemma 6. Let $<G,\vec{f}>$ and $<G',\vec{r}'>$ be the input and the output of Algorithm A3, respectively. Then, $<G,\vec{f}>$ is a YES instance of FF iff $<G',\vec{r}'>$ is a YES instance of RF.

Proof. Suppose $<G,\vec{f}>$ is a YES instance of FF. Since $r'_{xy} = 0$ for all

$(x,y)\notin A'$, each packet can be transmitted directly from its origin to its

destination. Let this be our routing rule for $<G',\vec{r}'>$. The resulting link

traffic rates satisfy $f'_{xy} = r'_{xy}$ for all $(x,y)\epsilon A'$.

Let S' be a transmission set of G' containing no links of the form

(a,y) or (x,a) for some node $a\epsilon N \subset N'$. Then $S' \cup \{(a_1,a_2)\}$ is a transmission

set of G'. In this way the transmission sets of G used by the TDMA scheme

under which $\vec{f}$ is feasible can be augmented to yield a TDMA scheme for G'

under which $\vec{f}'$ is feasible.

Suppose $<G',\vec{r}'>$ is a YES instance of RF. Since $R'_a + R'_{a_1} = 1$ and

$R'_a + R_{a_2} = 1$ for all $a\epsilon N \subset N'$, sending each packet directly from its origin

to its destination is the only routing rule under which $\vec{r}'$ can be feasible.

In that case $f_{xy} = r_{xy}$ for all $(x,y)\epsilon A'$, implying that $<G,\vec{f}>$ is a YES

instance of FF.

As the above proof suggests, if $<G,\vec{r}>$ is a YES instance of RF, then

there exists a stationary routing rule which determines the link traffic

rates and a TDMA scheme under which those rates are feasible. Therefore,

a concise certificate for a YES instance of RF is a set of routing variables

and a TDMA scheme satisfying the resulting link traffic rates. This com-
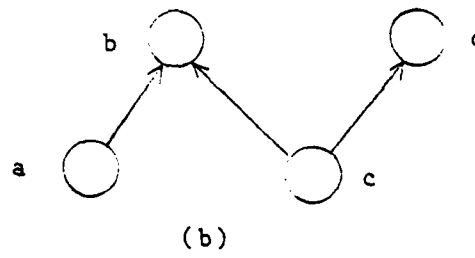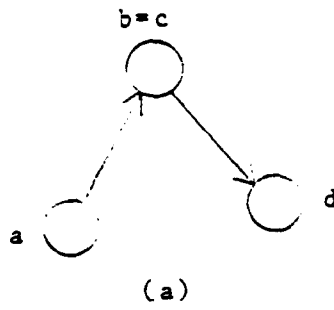
pletes the proof that RF is NP-complete.
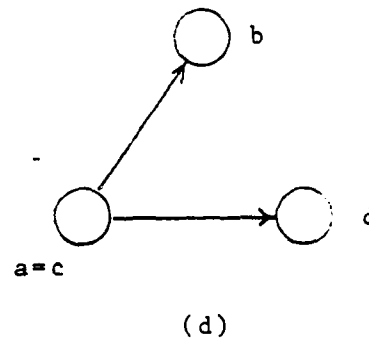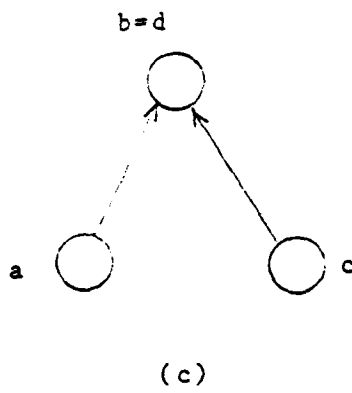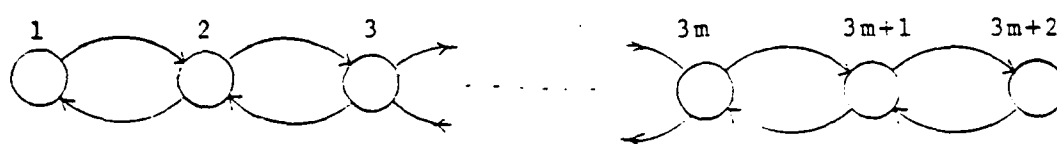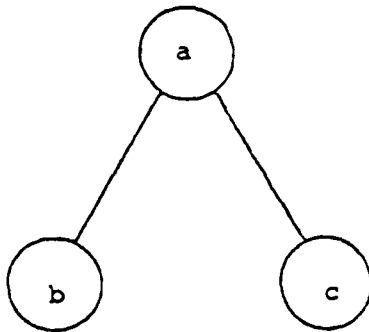
Fig. 1.
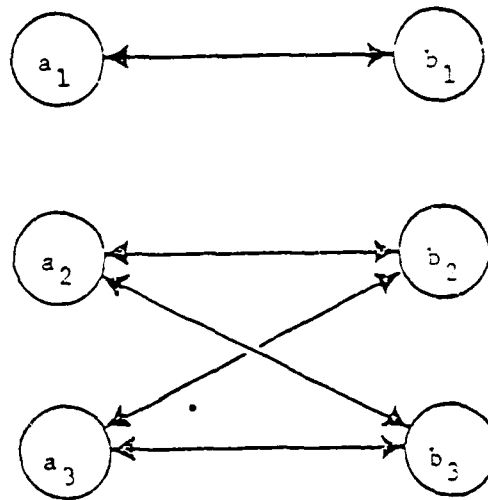
Situations in a PRN for which (c,d) conflicts with (a,b).

Fig. 2.

A chain PRN with 3m+2 nodes contains at least 2×3^m

maximal transmission vectors.(Use induction on m.)
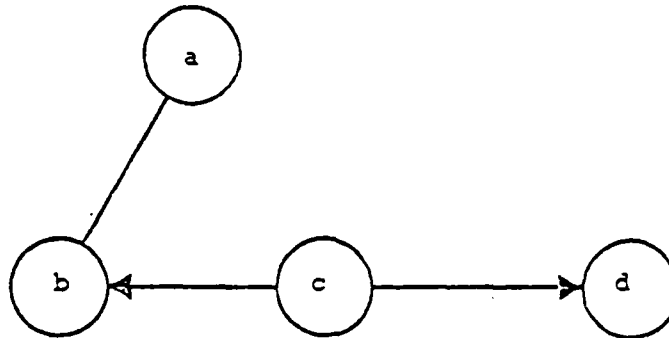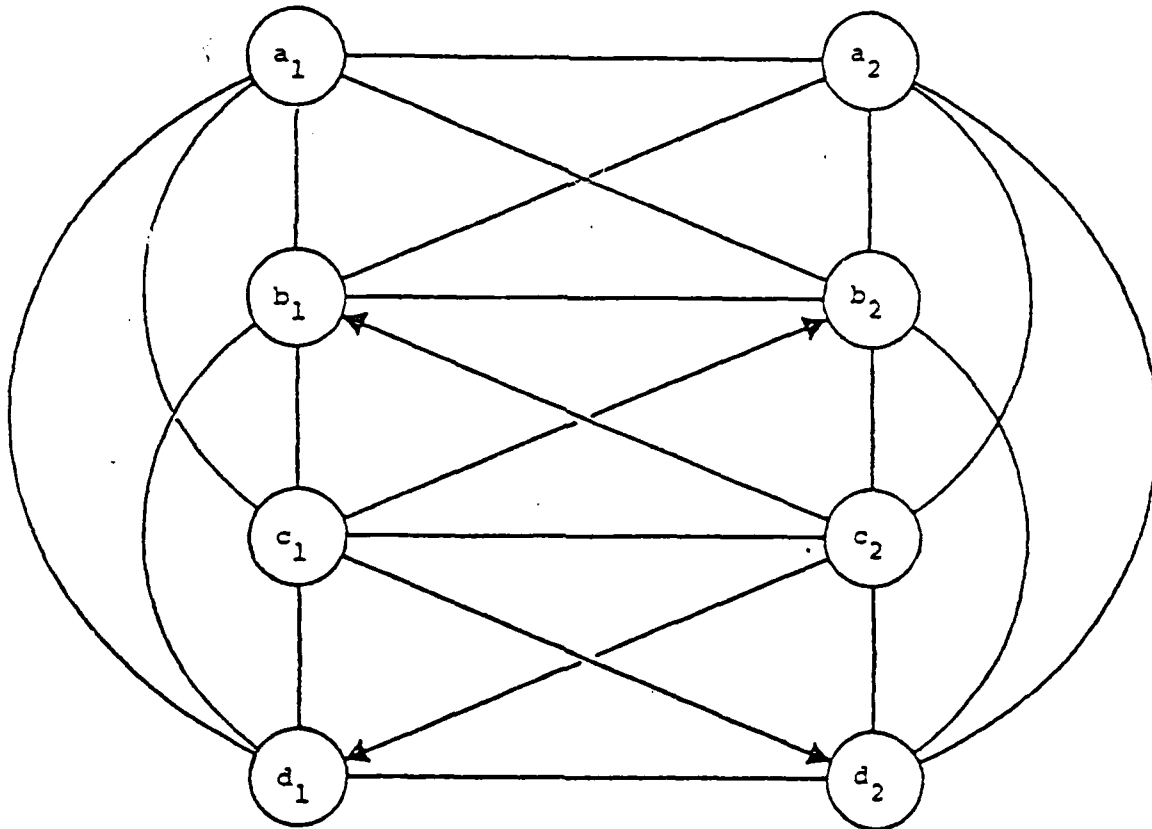
H = (V,E)

G = (N,A)

(a)

(b)

$$B = \{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}$$

Fig. A1.

(A link without an arrow stands for two oppositely directed
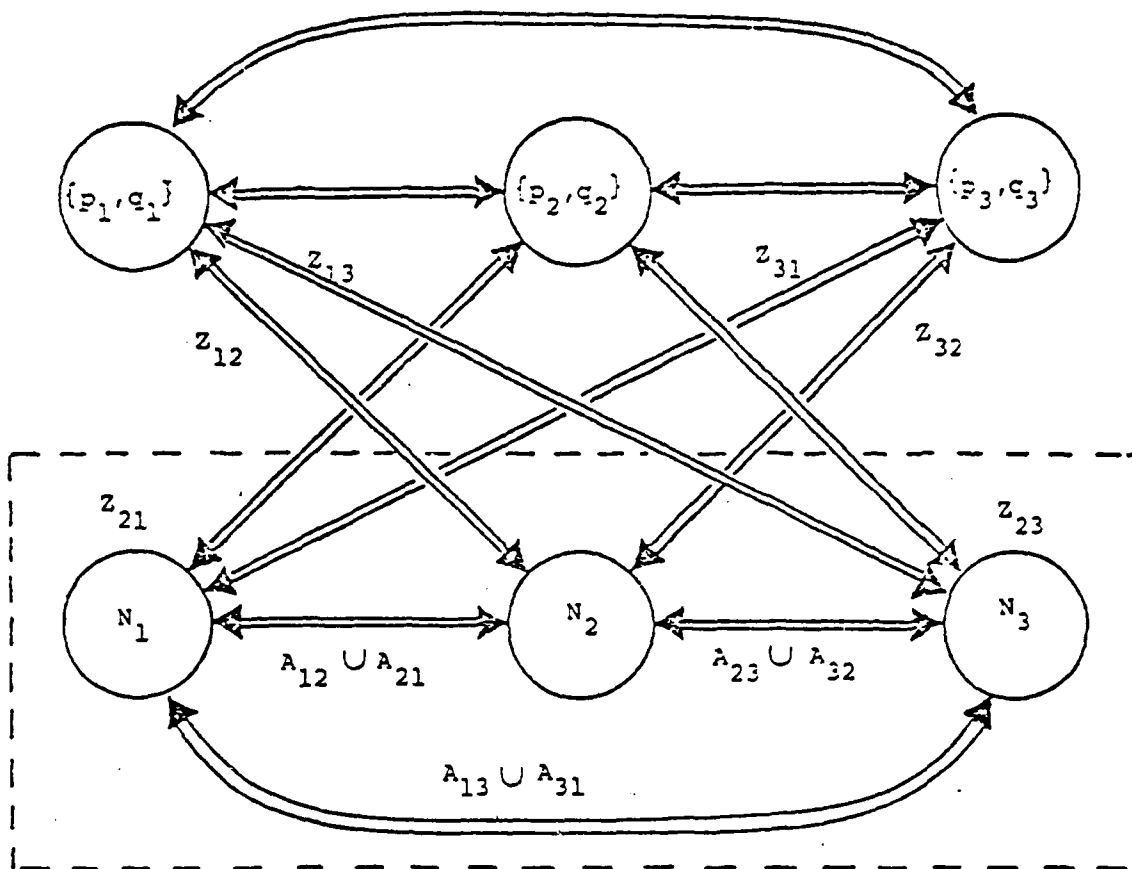links in this figure.)
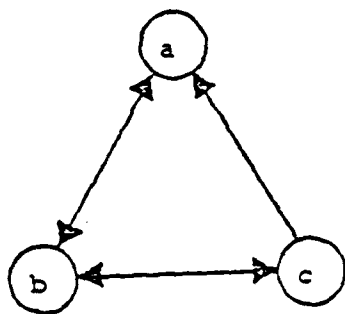

(a) $G = (N,A)$

(b) $G^2 = (N^2, A^2)$

Fig. A2.

$$G' = (N', A')$$

Fig. A3.

$$f_{ab} = 0.2$$
$$f_{ba} = 0.3$$
$$f_{bc} = 0.1$$
$$f_{cb} = 0$$
$$f_{ca} = 0.5$$

(a) $G = (N,A)$

r_{ab}	= 0.2	r_{ba}	= 0.3
r_{bc}	= 0.1	r_{cb}	= 0
r_{ca}	= 0.5	$r_{a_1 a_2}$	= 0
$r_{b_1 b_2}$	= 0.4	$r_{c_1 c_2}$	= 0.4

All other o-d rates are *zero*.

(b) $G' = (N', A')$

Fig. A4.